Group-Size Selection for a Parameterized Class of Predator-Prey Models

Jean-Pierre de la Croix and Magnus Egerstedt

***Abstract*— In this paper, we investigate whether we can qualitatively recover the appropriate group sizes for a team of predators by varying environmental and operational conditions. The result is a combination of biologically inspired analytical and algorithmic tools that not only establish guaranteed capture conditions, but also identify the number of predators needed for a successful capture. We implement the parameterized model on a team of mobile robots to validate that it is possible to generate a cooperative strategy that achieves capture with this model and these tools.**

I. INTRODUCTION

A number of multi-robot tasks involve finding, securing, and moving objects. These objects may be static as well as dynamic, and significant inspiration has been drawn from biology in general, and foraging and predator-prey models in particular, when designing cooperative behaviors. In fact, when designing cooperative capture strategies for multi-agent robotic systems (e.g., [1], [2], [6], and [14]), biological inspiration has played a key role. Nature provides us with numerous examples of how predators and prey interact.

One remarkable aspect of the different social foraging strategies found in nature is that they are highly diverse, both in terms of execution and the number of individuals partaking in the hunt. A cheetah, for example, is a solitary hunter who does not solicit assistance with the hunt [4]. Lions, on the other hand, rely less on speed and more on strategic geometric configurations with the use of a so-called "catcher's mitt" formation. Typically, three to five lionesses are involved in the hunt, with the dominant female taking the central position, while the remaining animals are spread out in the wing positions [4]. Bottle-nosed dolphins hunt in even larger groups, and one striking strategy employed is the so-called horizontal carousel, where up to 15 individuals encircle the prey and then gradually shrink the encirclement in order to capture the fish [10]. In this paper, we wish to make this informal observation concrete by devising models for cooperative multi-robot foraging strategies, where the number of participating predators emerges from the conditions surrounding the hunt.

In order to be able to predict how many predators are needed, we need models that are simple enough to analyze, yet expressive enough to allow for a parametrization that captures different foraging strategies. It should already be stressed at at this point, however, that our aim is *not* biomimicry, i.e., we make no statements about the biological validity of the proposed models. Instead, we simply draw inspiration from one of the more well-studied predator-prey interaction models: The lion, *Panthera leo*, is a top predator of the African Savannah while the Thomson's gazelle, *Eudorcas thomsonii*, is one of the lion's natural prey. Lions are social predators and will cooperatively hunt gazelles to feed. The gazelle uses its superior agility to evade the lions and, therefore, not every hunt is successful for the lions. In this paper, we produce parameterized interaction models inspired by the lions and gazelles (as described in [3], [4], [5], and [12]) to generate capture strategies for multi-agent robot teams and investigate the conditions needed for such a strategy to be successful.

A capture scenario is uninteresting from a cooperative vantage-point if a single predator has a clear advantage over its prey. A lion typically has a low rate of success hunting a gazelle on its own, because the gazelle has superior eye sight for detection and agility for evasion. As such, lions hunt cooperatively to increase their chance of success, and a natural question is how many lions are needed to capture a *particular* gazelle? We emphasize "a *particular* gazelle", because we consider each prey to be unique and we characterize its behavior in the presence of predators with a set of parameters. These parameters allow us to adjust the behavior of the prey, such that the predators can hunt a variety of strong, weak, brave, or skittish prey. Similarly, we define parameters that describe the hunting strategy of the predators, such as their speed and formation as a group during the hunt. Again, the parameters that describe the characteristics of the predators and prey are not to be taken as a faithful model of nature (in contrast to [13]), but rather as weak biological inspiration, allowing us to investigate whether nature-like capture strategies emerge from our mathematical formulation of the predator-prey interactions. Secondly, the models are generative in the sense that they are easily deployed on multi-robot systems (shown in Fig. 1). These models also allow us to analytically and algorithmically answer whether a capture strategy for a multi-agent robot team will be successful against a particular target. Third, very few results (for example, [7]) exist that relate the number of agents in a multi-agent team to the success at achieving the task at hand. This paper can thus be thought of as one particular attempt at addressing this previously neglected question concerning how large a team needs to be to achieve a particular task.

The key to characterizing capture strategies in multi-agent robot teams are the parameterized interaction dynamics. A common choice for cooperative multi-agent robot applications is a weighted consensus-type equation, discussed in

J.-P. de la Croix and M. Egerstedt are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30308, USA {jdelacroix,magnus}@gatech.edu

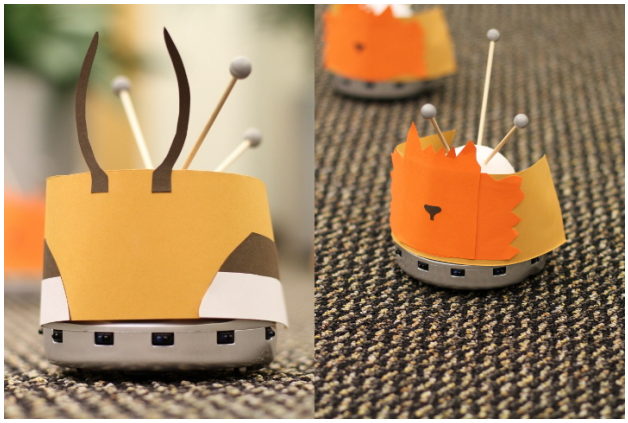

Fig. 1. Mobile robots as surrogates for the prey (gazelles) and predators (lions).

detail in [8] and [9]. Both cohesion and separation can be achieved simultaneously with such a formulation. In this paper, we start with a more general interaction model that is later made more concrete using the weighted consensus equation originally proposed in [5].

Given our choice of parameterized interaction dynamics, we can analytically compute the velocity needed for a single predator to capture a particular prey. Similarly, if a group of three or more predators is hunting in a semicircular formation (much like in nature, see [3], [4], and [6]), then we can derive conditions for a successful capture in terms of the predator and prey parameters. In fact, we can show that slower predators can capture the same prey if more predators participate in the hunt. Since we are ultimately interested in answering the original question of *how many robots does it take to capture a particular target?*, we provide an algorithmic solution to find the minimum number of predators needed to successfully capture the prey. Lastly, we experimentally demonstrate on mobile robot platforms that successful capture strategies emerge from the parameterized interaction dynamics that are somewhat akin to the strategies observed when lions hunt gazelles in nature.

II. Geometric Hunting Strategies

There exist a great variety of dynamics that can characterize the interactions between agents in a team. We are interested in a particular set of interaction dynamics, namely predator-prey interactions. These interactions are unique in that the agents are separated into two classes: one class of agents, the predators, is motivated to capture the other class of agents, the prey. As already stated, we are weakly inspired by the predator-prey interactions between lions and gazelles, where the predators hunt as a team in a strategic geometric configuration (i.e., a formation) to overcome the evasive abilities of the prey. Fig. 2 illustrates one such formation for a team of N predators. The dominant predator (to borrow from the lion terminology) in the team takes the central position on the x-axis, while the remaining predators are spread symmetrically to either side in the "wing" positions. Each predator is separated from its nearest team member by

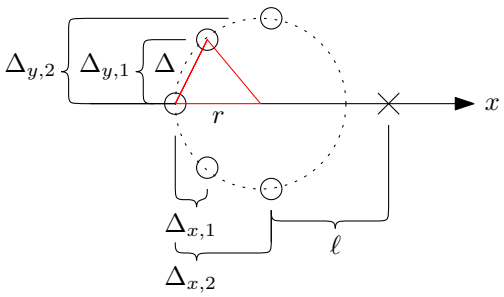

Fig. 2. A group of $N = 5$ predators (circles) hunting a single prey (cross).

a distance Δ, and the predators can be thought of being uniformly distributed on a circle of radius r centered on the x-axis. Furthermore, ℓ is the distance separating the outermost wing predators and the prey along the x-axis (denoted by a cross in the figure).

Counting from the center predator, the zeroth level is the center predator, the first set of wing predators is the first level, the next set of wing predators is the second level, and so on. There are $m = (N-1)/2$ levels of wing predators. And, for each predator, we can compute its x and y position in the formation with respect to the center predator:

$$\begin{aligned} \Delta_{x,k} &= r - r\cos\left(2k\sin^{-1}\left(\frac{\Delta}{2r}\right)\right) \\ |\Delta_{y,k}| &= r\sin\left(2k\sin^{-1}\left(\frac{\Delta}{2r}\right)\right), \end{aligned} \tag{1}$$

where $k = \{0, 1, \ldots, m\}$, and where a k-th level wing predator is located at $(\Delta_{x,k}, \Delta_{y,k})$ with respect to the center predator. We can also compute how far a predator at the k-th level is from the prey along the x-axis, i.e.,

$$\lambda_k(\ell) = \ell + \Delta_{x,m} - \Delta_{x,k}. \tag{2}$$

Then, the total distance separating the prey and a predator at the k-th level is,

$$\delta_k(\ell) = \sqrt{\lambda_k(\ell)^2 + \Delta_{y,k}^2}. \tag{3}$$

The center predator is the zeroth level, such that $\lambda_0(\ell) = \delta_0(\ell) = \ell + \Delta_{x,m}$, since $\Delta_{x,0} = \Delta_{y,0} = 0$.

Now that we have described the initial *geometry* of the hunt, we are ready to describe its *dynamics*. The predators hunt together along the x-axis in the direction of the prey with a constant, scalar velocity v and without deviating from their configuration. If $p_l(t) \in \mathbf{R}^2$ is the position of a particular predator at time t (where we somewhat facetiously use the subscript l to denote "lion"), then

$$\dot{p}_l(t) = \begin{bmatrix} \dot{x}_l(t) \\ \dot{y}_l(t) \end{bmatrix} = \begin{bmatrix} v \\ 0 \end{bmatrix}. \tag{4}$$

The dynamics of the prey with respect to a particular predator at the k-th level are encoded by a continuous, monotonically non-increasing function $\Gamma : \mathbf{R} \to \mathbf{R}$, such that it starts off at some value $\Gamma_{\max}$ and ends up at 0 after a finite interval, i.e.,

$$\Gamma(\psi) = \begin{cases} \Gamma_{\max} & \text{if } \psi \leq \Delta_{\max} \\ 0 & \text{if } \psi \geq \Delta_d \end{cases}, \tag{5}$$

where $\Delta_{\max}$ is the distance from a predator when the prey will start evading with maximum effort, and Δ_d is the distance from a predator when the prey can detect the predator, and where we assume that $\Delta_{\max} < \Delta_d$.

If we denote the contribution to the prey dynamics of a predator at the k-th level as

$$\Gamma_k(\ell) = \Gamma(\delta_k(\ell)) = \Gamma\left(\sqrt{\lambda_k(\ell)^2 + \Delta_{y,k}^2}\right), \quad (6)$$

then we can write down the full dynamics of the prey for the N predator hunt. If $p_g(t) = [x_g(t), y_g(t)]^T \in \mathbf{R}^2$ is the position of the prey at time t (where we use the subscript g to denote "gazelle"), then

$$\dot{x}_g = \Gamma_0(\ell) + 2\sum_{k=1}^{m} \frac{\Gamma_k(\ell)\lambda_k(\ell)}{\sqrt{\lambda_k(\ell)^2 + \Delta_{y,k}^2}}, \quad (7)$$

and $\dot{y}_g = 0$. The symmetric configuration of wing predators during the hunt ensures that any component along the y-axis is always equal to zero.

For the purpose of generality we will use Equation (7), but we will ground our results on a slightly modified version of an actual swarm interaction dynamic–originally proposed in [5]. Under this model the contribution from a predator located at the k-th level is

$$\Gamma_k^{'}(\ell) = \begin{cases} \beta\sqrt{\frac{\gamma}{2e}} & \lambda_k(\ell) \leq \sqrt{\frac{\gamma}{2}} \\ \beta\lambda_k(\ell)e^{-\frac{\delta_k(\ell)^2}{\gamma}} & \sqrt{\frac{\gamma}{2}} < \lambda_k(\ell) < \infty \\ 0 & \lambda_k(\ell) \geq \infty \end{cases}, \quad (8)$$

such that the full dynamics of the prey for a N predator hunt can be written as

$$\dot{x}_g = \Gamma_0^{'}(\ell) + 2\sum_{k=1}^{m} \Gamma_k^{'}(\ell), \quad (9)$$

and $\dot{y}_g = 0$, where β, and γ are certain behavioral parameters. In fact, the interaction dynamics described in [5] defines an additional parameter, which dictates whether an agent is drawn towards another agent. Some predators, for example the anglerfish *Lophius piscatorius*, are able to attract their prey. However, lions and most predators do not hunt in this manner, and therefore we exclude this phenomenon from the model. Instead we focus our attention on the two parameters β and γ. These two parameters together characterize with how much effort the prey attempts to escape from a predator as the distance to the predator decreases. The maximum effort with which the prey evades a predator is $\beta\sqrt{\frac{\gamma}{2e}}$, where as its maximum effort is captured by $\Gamma_{\max}$ in Equation (5). Moreover, γ parameterizes how close the predators can approach the prey before it evades with maximum effort. This distance is captured by $\Delta_{\max}$ in (5), while Δ_d in (5) captures the distance at which the prey does not detect the predators.

Although all prey are scared of predators and will attempt to escape them, we do not need to impose that different prey are alike. Therefore, we have two prey parameters that can vary: β and γ (or $\Gamma_{\max}$ and $\Delta_{\max}$). We also consider three additional parameters that characterize the formation and dynamics of the predators. A group of predators will move together in a formation parameterized by Δ and r with a constant velocity v, which gives us three additional "knobs" through which the hunt dynamics can be characterized, namely v, Δ, and r.

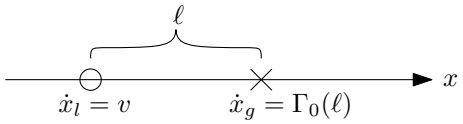

Fig. 3. A single predator (circle) hunting a single prey (cross).

A. One Predator

The first predator-prey scenario is a single predator hunting a single prey. Fig. 3 shows the predator-prey configuration we consider in the scenario. This particular configuration, which places the predator and the prey on the line $y = 0$, allows us to only consider:

$$\dot{x}_g = \Gamma_0(\ell) = \Gamma(\ell). \quad (10)$$

Since we are interested in hunting, let us consider that a capture occurs when ℓ, the distance separating the predator from the prey, is equal to zero. We want to find the velocity of the predator, v, that guarantees that capture occurs.

Theorem 1: If $\Gamma_{\max}$ is the maximal effort the prey can use to evade the predator, then the predator is able to the capture the prey if and only if the predator's velocity is $v > \Gamma_{\max}$.

This result is not particularly surprising. What it simply says is that the predator's hunt velocity v must be greater than the prey's maximal evasion velocity, which is, for example, how solitary Cheetahs hunt. However, the proof of this statement will be useful for more complex situations and, as such, we do include it below:

Proof: Let ℓ^* be the distance separating the predator and the prey that maximizes the derivative $\dot{\ell}$, where

$$\dot{\ell} = \dot{x}_g - \dot{x}_l = \Gamma(\ell) - v. \quad (11)$$

Then the predator is able to capture the prey if and only if $\dot{\ell}(\ell^*) < 0$. By construction of $\Gamma(\psi)$, we know that $\Gamma(\ell^*)$ is maximized when $\ell^* \leq \Delta_{\max}$. Therefore,

$$\dot{\ell}(\ell^*) = \Gamma(\ell^*) - v < 0, \quad (12)$$

which means that

$$\Gamma_{\max} - v < 0 \;\Leftrightarrow\; \Gamma_{\max} < v, \quad (13)$$

which means that given ℓ^*, the predator can capture the prey if $v > \Gamma_{\max}$. ■

If we consider the predator-prey interaction dynamic defined in (9), we can apply this theorem to find the velocity v of the predator that is guaranteed to capture a prey parameterized by β and γ. The configuration shown in Fig. 3 allows us to formulate the prey dynamics as:

$$\dot{x}_g = \beta\ell e^{-\frac{\ell^2}{\gamma}}.$$

We can detect a capture by checking if $\dot{\ell}(\ell^*)$ is negative, where $\dot{\ell}$ is now defined in the following way:

$$\dot{\ell} = \dot{x}_g - \dot{x}_l = \beta \ell e^{-\frac{\ell^2}{\gamma}} - v.$$

We solve for ℓ^*, the value of ℓ that maximizes $\dot{\ell}$, by setting the derivative of $\dot{\ell}$ with respect to ℓ to zero and simplifying:

$$\begin{aligned}\frac{\partial \dot{\ell}}{\partial \ell} = \beta e^{-\frac{\ell^2}{\gamma}} - \frac{2\beta\ell^2}{\gamma} e^{-\frac{\ell^2}{\gamma}} &= 0 \\ \left(1 - \frac{2\ell^2}{\gamma}\right) \beta e^{-\frac{\ell^2}{\gamma}} &= 0 \\ \left(1 - \frac{2\ell^2}{\gamma}\right) &= 0 \\ \sqrt{\frac{\gamma}{2}} &= \ell^*\end{aligned}$$

We plug ℓ^* back into the equation for $\dot{\ell}$ and set this equation equal to less than zero to solve for v:

$$\dot{\ell}(\ell^*) = \beta \sqrt{\frac{\gamma}{2}} e^{-\frac{\sqrt{\frac{\gamma}{2}}^2}{\gamma}} - v < 0,$$

i.e.,

$$\beta \sqrt{\frac{\gamma}{2e}} < v. \tag{14}$$

As such, the predator can capture the prey if and only if $v > \beta\sqrt{\frac{\gamma}{2e}}$, when the maximal effort of the prey, $\Gamma_{\max} = \beta\sqrt{\frac{\gamma}{2e}}$, is held on the interval $\ell \leq \sqrt{\frac{\gamma}{2}}$.

Let us briefly examine a concrete example. Fig. 4 is a graph of $\dot{\ell}$ as a function of $\ell \in [0, 2]$. The maximum of $\dot{\ell}$ occurs at $\ell^* = \sqrt{\frac{\gamma}{2}} \approx 0.274$, which agrees with our derivation. We also show two cases for the predator's velocity: 0.1 m/s and 0.3 m/s. First, if the predator's velocity is 0.1 m/s, then the maximum of $\dot{\ell}$ is above the solid line, meaning that $\dot{\ell}(\ell^*) > 0$ and the predator is not fast enough to capture the prey,

$$v = 0.1 < 1.5\sqrt{\frac{0.15}{2e}} \approx 0.249.$$

In the second case, the predator's velocity is 0.3 m/s, such that $\dot{\ell}(\ell^*) < 0$, and the predator is able to capture the prey, since $v = 0.3 > \Gamma_{\max} = 0.249$.

B. Three Predators

The previous section gave us an expression for the velocity required for a single predator to capture a prey. However, predators are typically not as fast as the prey, so they hunt cooperatively in groups to increase their chance of success. Let us consider a configuration with three predators, as illustrated in Fig. 5. We want to derive a similar capture condition on the velocity of the predators as in the previous case; however, in this scenario $\ell = 0$ implies that capture is achieved when the wing predators at the m-th level achieve "crossover", i.e. these leading wing predators pass the prey. We first show that it is indeed sufficient to just consider the crossover condition and moreover note that this condition

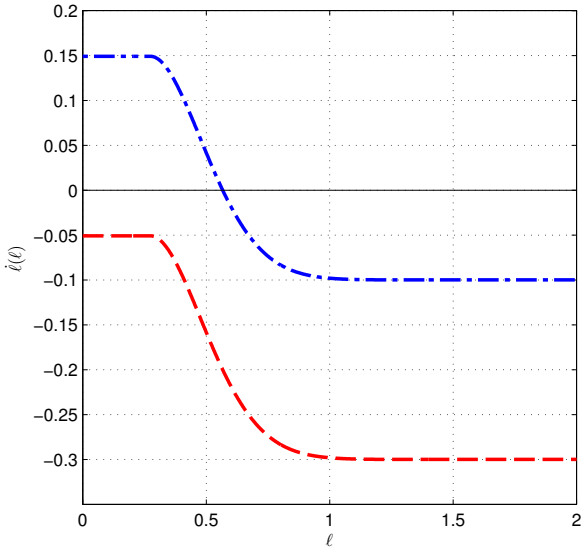

Fig. 4. $\beta = 1.5$, $\gamma = 0.15$; Dashed-dotted: $v = 0.1$ m/s; Dashed: $v = 0.3$ m/s.

is very much in line with the result on cooperative pursuit-evasion games that rely of driving the evader into the convex hull spanned by the pursuers, as was done in [11].

Theorem 2: If at time t_c, the two leading wing predators reach the prey with respect to the x-axis, i.e. $x_l^w(t_c) = x_g(t_c)$, then there exists a strategy which guarantees capture after this "crossover" event.

Proof: If crossover occurs at time t_c, then this event implies that $0 < \dot{x}_g(t_c) \leq v$. We can expand $\dot{x}_g$, such that

$$\dot{x}_g = \dot{x}_g^w + \dot{x}_g^{L\setminus w}, \tag{15}$$

where $\dot{x}_g^w$ is the contribution to $\dot{x}_g$ from the leading wing predators, and $\dot{x}_g^{L\setminus w}$ is the contribution from all predators excluding the leading wing predators.

Since $\dot{x}_g^w(t_c)$, the contribution from the leading wing predators, is zero when they are at the crossover point, we can write $\dot{x}_g(t_c) = \dot{x}_g^{L\setminus w}(t_c)$. At time t_c all predators except the leading wing predators stop, such that $\dot{x}_l^{L\setminus w}(t_c) = 0$. By continuity, there exists $\Delta T > 0$, such that

$$\dot{x}_g(t) > 0, \, \forall t \in (t_c, t_c + \Delta T]. \tag{16}$$

But on this interval, $\dot{x}_g^{L\setminus w}(t) < \dot{x}_g^{L\setminus w}(t_c) = \dot{x}_g(t_c)$ and since $\dot{x}_l^w(t) = v$, we have $\dot{x}_g^{L\setminus w}(t) < 0$. In other words, the leading wing predators pass the prey and contribute a push towards the rest of the predators.

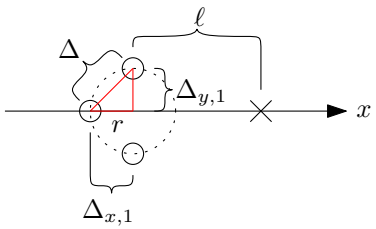

Fig. 5. A group of three predators (circles) hunting a single prey (cross).

Let us pick $t' \in (t_c, t_c + \Delta T]$ and let $v' = \dot{x}_g(t') < v$. Since $\dot{x}_g(t) \leq \dot{x}_g(t')\ \forall t \geq t'$ and $v > v'$, there exists a T such that

$$|x_g(T) - x_l^w(T)| > \frac{v' y_l^w(t_c)}{v}, \tag{17}$$

where $y_l^w(t_c)$ is the position of the leading wing predators in the y-direction at time t_c. Then, at time T, a possible capture strategy is to first stop the motion of the leading wing predators in the x-direction, such that $\dot{x}_l^w(T) = 0$. Next, let these wing predators converge on the x-axis, such that $y_l^w(t) \to 0, t > T$. Once they have converged, $y_l^w(t) = 0$ and the leading wing predators can regroup with the rest of the predators, such that $|x_l^{L\setminus w}(t) - x_l^w(t)| \to 0$, $t > T$ and capture is achieved. ■

We are now justified in simply using the event that the leading wing predators reach the crossover point, $\ell = 0$, as a condition for guaranteed capture.

Let us return to the three predator scenario as depicted in Fig. 5. We define the derivative, $\dot{\ell}(\ell)$, as

$$\begin{aligned}\dot{\ell}(\ell) &= \Gamma_0(\ell) + 2\frac{\Gamma_1(\ell)\lambda_1(\ell)}{\sqrt{\lambda_1(\ell)^2 + \Delta_{y,1}^2}} - v' \\ &= \Gamma(\ell + \Delta_{x,1}) + 2\frac{\Gamma\left(\sqrt{\ell^2 + \Delta_{y,1}^2}\right)}{\sqrt{\ell^2 + \Delta_{y,1}^2}}\ell - v'.\end{aligned} \tag{18}$$

We want to find a configuration of predators, Δ and $v' < v$, such that we can capture a single prey with slower predators.

Theorem 3: There exists $\Delta_y > 0$, such that captured is guaranteed when $v' < v$, if and only if $\Delta_x > \Delta_{\max}$.

Proof: For capture to be possible with three predators, we need to satisfy:

$$\Gamma(\ell^* + \Delta_x) + 2\frac{\Gamma\left(\sqrt{(\ell^*)^2 + \Delta_y^2}\right)}{\sqrt{(\ell^*)^2 + \Delta_y^2}}\ell^* < v', \tag{19}$$

where ℓ^* maximizes (18), $\Delta_x = \Delta_{x,1}$, and $\Delta_y = \Delta_{y,1}$. We also want to satisfy that $v' < v$, therefore:

$$\Gamma(\ell^* + \Delta_x) + 2\frac{\Gamma\left(\sqrt{(\ell^*)^2 + \Delta_y^2}\right)}{\sqrt{(\ell^*)^2 + \Delta_y^2}}\ell^* < \Gamma_{\max}. \tag{20}$$

The contribution from the center predator has to satisfy

$$\Gamma(\ell + \Delta_x) < \Gamma_{\max},$$

otherwise it is impossible to satisfy (20). Its contribution is less than $\Gamma_{\max}$ for all $\ell \geq 0$ if and only if $\Delta_x > \Delta_{\max}$ by construction of $\Gamma(\psi)$.

Suppose $\Delta_x > \Delta_{\max}$, then we can rewrite and rearrange (20) as,

$$\begin{aligned}\Gamma_{\max}(1-\epsilon) + 2\frac{\Gamma\left(\sqrt{(\ell^*)^2 + \Delta_y^2}\right)}{\sqrt{(\ell^*)^2 + \Delta_y^2}}\ell^* &< \Gamma_{\max} \\ \frac{\Gamma\left(\sqrt{(\ell^*)^2 + \Delta_y^2}\right)}{\sqrt{(\ell^*)^2 + \Delta_y^2}}\ell^* &< \frac{\epsilon}{2}\Gamma_{\max},\end{aligned} \tag{21}$$

where $\epsilon > 0$ and $\epsilon \in (0, 1]$. We can apply one more bound to (21),

$$\begin{aligned}\frac{\Gamma\left(\sqrt{(\ell^*)^2 + \Delta_y^2}\right)}{\sqrt{(\ell^*)^2 + \Delta_y^2}}\hat{\ell} \leq \frac{\Gamma_{\max}}{\sqrt{(\ell^*)^2 + \Delta_y^2}}\ell^* &< \frac{\epsilon}{2}\Gamma_{\max} \\ \frac{\ell^*}{\sqrt{(\ell^*)^2 + \Delta_y^2}} &< \frac{\epsilon}{2}\end{aligned} \tag{22}$$

We can satisfy (22) by picking Δ_y sufficiently large, such that the inequality holds independent of ℓ^*. ■

Corollary 1: Given that $\Delta_y > 0$ and $\Delta_x > \Delta_{\max}$, then $\Delta > \Delta_{\max}$ is a lower bound on Δ that must hold for capture to be possible when $v' < \Gamma_{\max} < v$ and Δ_y is sufficiently large.

Returning to the predator-prey dynamics defined in (9), if $\Delta_x > \sqrt{\frac{\gamma}{2}}$, then three predators are able to cooperatively capture a prey at a velocity v' that is smaller than $\beta\sqrt{\frac{\gamma}{2}}$, i.e the velocity v that is required by a single predator to capture the same prey.

The expression for $\dot{\ell}$ with the addition of two wing predators to the single predator is,

$$\dot{\ell} = \beta(\ell + \Delta_x)e^{-\frac{(\ell+\Delta_x)^2}{\gamma}} + 2\beta\ell e^{-\frac{(\ell^2+\Delta_y^2)}{\gamma}} - v' \tag{23}$$

or

$$\dot{\ell} = \Lambda^c + \Lambda^w - v',$$

where,

$$\Lambda^c = \beta(\ell + \Delta_x)e^{-\frac{(\ell+\Delta_x)^2}{\gamma}}$$

is the contribution from the center predator, and

$$\Lambda^w = 2\beta\ell e^{-\frac{(\ell^2+\Delta_y^2)}{\gamma}}$$

is the contribution from the wing predators.

First, the contribution from the center predator, Λ^c, attains its maximum in the region $\ell \geq 0$ when $\ell = 0$, i.e. the maximum push from the center predator occurs when the wing predators achieve crossover.

$$\Lambda^c_{\max} = \Lambda^c\left(\ell = 0\right) = \beta\Delta_x e^{-\frac{\Delta_x^2}{\gamma}}.$$

Second, the contribution from the wing predators, Λ^w is maximized when $\ell = \sqrt{\frac{\gamma}{2}}$.

$$\Lambda^w_{\max} = \Lambda^w\left(\ell = \sqrt{\frac{\gamma}{2}}\right) = 2\beta\sqrt{\frac{\gamma}{2e}}e^{-\frac{\Delta_y^2}{\gamma}}$$

As in the case of the single predator, we again want $\dot{\ell} < 0$, such that $\Lambda^c + \Lambda^w < v'$, and since $\Lambda^c_{\max} + \Lambda^w_{\max} > \Lambda^c + \Lambda^w$ and $v' < v$:

$$\Lambda^c_{\max} + \Lambda^w_{\max} < v' < v$$

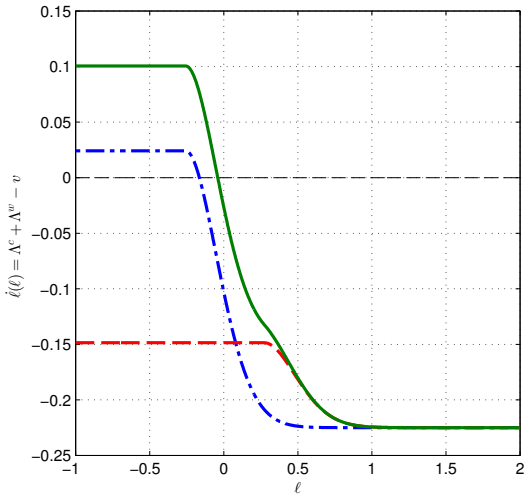

Fig. 6. $\beta = 1.5$, $\gamma = 0.15$, $v = 0.225$, $\Delta = 0.75$, $r = 0.75/(2\sin(\pi/8))$. The dashed and dashed-dotted lines are the components contributed to $\dot{\ell}(\ell)$ by the center and wing predators minus v respectively. The solid line is $\dot{\ell}(\ell)$.

If $\Delta_x > \sqrt{\frac{\gamma}{2}}$, then using this inequality and $v > \sqrt{\frac{\gamma}{2e}}$, we can show the following:

$$\begin{aligned}
\beta\Delta_x e^{-\frac{\Delta_x^2}{\gamma}} + 2\beta\sqrt{\frac{\gamma}{2e}}e^{-\frac{\Delta_y^2}{\gamma}} &< v \\
\beta\Delta_x e^{-\frac{\Delta_x^2}{\gamma}} + 2e^{-\frac{\Delta_y^2}{\gamma}}v &< v \\
(1-\epsilon)v + 2e^{-\frac{\Delta_y^2}{\gamma}}v &< v \\
2e^{\frac{-\Delta_y^2}{\gamma}}v &< \epsilon v \\
e^{\frac{-\Delta_y^2}{\gamma}} &< \frac{\epsilon}{2}
\end{aligned}$$

There exists a Δ_y sufficiently large for which this inequality will hold, since $\epsilon > 0$ and $\epsilon \in (0, 1]$. As a consequence, there exists Δ_y large enough, such that $v > v'$ holds.

Let us inspect $\dot{\ell}$ from (23) in Fig. 6 and check whether we can say something equally useful as in the single predator case. We choose the parameters $\beta = 1.5$, $\gamma = 0.15$, $v = 0.225$, $\Delta = 0.75$, and $\phi = 0.75/(2\sin(\pi/8))$ for which the two conditions hold. The dashed line is the component of $\dot{\ell}$ that is contributed by the center predator, while the dashed-dotted line is the contribution from the two wing predators. The solid line is the total contribution to $\dot{\ell}$ from all predators. A velocity of 0.225m/s is not sufficient for a single predator to capture the prey (i.e., $\dot{\ell}(\ell) > 0$ and recall that the contribution from the center predator is offset by Δ_x); however, it is sufficient for three predators to cooperatively capture this particular prey. The dashed-dotted line has a negative maximum; however, it is unclear whether there exists a capture strategy for two predators passing the prey to either side that implies capture; therefore, we will consider only the cases where we have a center predator.

C. N Predators

Let us return to the configuration with N predators and a single prey, as illustrated in Fig. 2, since we are originally interested in the question of how many predators it takes to capture a particular prey. We want to show that a group of N predators can capture a prey at a velocity v' that is less than the velocity v required for a single predator to capture the same prey.

Theorem 4: There exists a $\Delta_y = \min\{\Delta_{y,1}, \ldots, \Delta_{y,m}\}$, such that $v' < v$ and capture is guaranteed, if and only if $\Delta_{x,m} > \Delta_{\max}$.

Proof: For capture to be possible with N predators, we need to satisfy

$$\Gamma_0(\ell^*) + 2\sum_{k=1}^{m} \frac{\Gamma_k(\ell^*)\lambda_k(\ell^*)}{\sqrt{\lambda_k(\ell^*)^2 + \Delta_{y,k}^2}} < v' \tag{24}$$

where ℓ^* maximizes $\dot{\ell}(\ell)$. We also want to satisfy that $v' < v$, therefore:

$$\Gamma_0(\ell^*) + 2\sum_{k=1}^{m} \frac{\Gamma_k(\ell^*)\lambda_k(\ell^*)}{\sqrt{\lambda_k(\ell^*)^2 + \Delta_{y,k}^2}} < \Gamma_{\max} \tag{25}$$

The contribution from the center predator has to satisfy $\Gamma_0 < \Gamma_{\max}$, otherise it is impossible to satisfy (25). Its contribution is less than $\Gamma_{\max}$ for all $\ell \geq 0$ if and only if $(r + \Delta_{x,m}) > \Delta_{\max}$ by construction of $\Gamma(\psi)$.

$$\begin{aligned}
\Gamma_{\max}(1-\epsilon) + 2\sum_{k=1}^{m} \frac{\Gamma_k(\ell^*)\lambda_k(\ell^*)}{\sqrt{\lambda_k(\ell^*)^2 + \Delta_{y,k}^2}} &< \Gamma_{\max} \\
\sum_{k=1}^{m} \frac{\Gamma_k(\ell^*)\lambda_k(\ell^*)}{\sqrt{\lambda_k(\ell^*)^2 + \Delta_{y,k}^2}} &< \frac{\epsilon}{2}\Gamma_{\max} \\
\sum_{k=1}^{m} \frac{\Gamma_{\max}\lambda_k(\ell^*)}{\sqrt{\lambda_k(\ell^*)^2 + \Delta_{y,k}^2}} &< \frac{\epsilon}{2}\Gamma_{\max} \\
\sum_{k=1}^{m} \frac{\lambda_k(\ell^*)}{\sqrt{\lambda_k(\ell^*)^2 + \Delta_{y,k}^2}} &< \frac{\epsilon}{2} \\
\sum_{k=1}^{m} \frac{\lambda_k(\ell^*)}{\sqrt{\lambda_k(\ell^*)^2 + \Delta_{y}^2}} &< \frac{\epsilon}{2}
\end{aligned} \tag{26}$$

where $\Delta_y = \min\{\Delta_{y,1}, \ldots, \Delta_{y,m}\}$ and $\epsilon \in (0, 1]$. ■

We would also like to show that if there is a configuration of N predators that can capture a prey with a velocity of v', then we can also capture a prey with some configuration of $N + 2$ predators at a slower velocity $v'' < v'$.

Corollary 2: There exists a $\Delta'_y = \min\{\Delta'_{y,1}, \ldots, \Delta'_{y,m}\}$, such that $v'' < v'$ if and only if $\Delta'_{x,m} > \Delta_{\max}$.

Proof: We can pick a $\Delta'_y = \min\{\Delta'_{y,1}, \ldots, \Delta'_{y,m}\}$ sufficiently large, such that

$$\sum_{k=1}^{m+1} \frac{\lambda_k(\ell'^*)}{\sqrt{\lambda_k(\ell'^*)^2 + \Delta'^2_y}} < \sum_{k=1}^{m} \frac{\lambda_k(\ell^*)}{\sqrt{\lambda_k(\ell^*)^2 + \Delta_y^2}} \tag{27}$$

where ℓ^* maximizes $\dot{\ell}(\ell)$ for the N predator hunt and ℓ'^* maximizes $\dot{\ell}(\ell)$ for the $N + 2$ predator hunt. Satisfying this inequality implies that $N + 2$ predators can capture the prey at a velocity $v'' < v'$. ■

III. Algorithm

We laid down the ground work in the previous section to answer the question of how many predators are needed to capture a particular prey. Specifically, we proved that there exists Δ_y that guarantees that a group of predators can capture a prey under certain conditions (characterized by a set of parameters). For the purposes of the proofs, we have been very conservative with the bounds to show that Δ_y can be made sufficiently large to satisfy the inequalities and thus guarantee capture. Practically speaking, Δ_y can be reasonable (and not necessarily arbitrarily large) depending on the predator and prey parameters selected.

Suppose we are interested in finding the minimum number of robotic "predators" needed to capture a specific prey parameterized by $\Gamma_{\max}$ and $\Delta_{\max}$. The predators are parameterized by v, Δ, r, and T. One way of checking the minimum number of such predators needed to capture this prey can be computed using the following algorithm:

TABLE I

HUNTING($\Gamma_{\max}, \Delta_{\max}, v, N_{\max}, \Delta, r, T$)

Input: Prey parameters $\Gamma_{\max}$ and $\Delta_{\max}$; predator parameters v, Δ, r, and T
Output: The minimum N (if it exists) for the given parameters
  for all $m = (N_0 - 1)/2$ to $(N_{\max} - 1)/2$ **do**
    if $(r - \Delta_{x,m}) > \Delta_{\max}$ **then**
      $[t, \ell(t)] \leftarrow$ `ODE45(@dynamics,` $[0, T]$, $\ell_0)$
      if $\exists t$, $\ell(t) < 0$ **then**
        $N_{\min} \leftarrow 2m + 1$
        return
      end if
    end if
  end for

T is a new parameter that captures how long the predators will attempt to hunt before running out of energy. Up until now we have assumed $T = \infty$, but we need T to be finite for the algorithm to terminate. $N_{\max}$ is odd and represents the maximum number of predators that can be recruited for the hunt. Since T and $N_{\max}$ are finite, this algorithm may terminate with no successful strategy even if one exists if T or $N_{\max}$ were larger. This limitation is reasonable when these strategies are deployed on robots, because we have a finite amount of time before battery power is expended, or we can only deploy a finite number of robots in the hunt.

IV. Experiment

We validate that is possible to generate a cooperative strategy with the proposed parameterized model that achieves capture by performing an experiment involving robotic "lions" hunting a robotic "gazelle". A differential drive mobile robot platform is used as a robotic surrogate for real predators and prey. These robotic gazelle and lions are provided with positional data from a motion capture system, which allows them to compute the inter-agent distances needed in the dynamics. A low-level controller converts the desired predator and prey motions into the appropriate differential drive velocities needed to actuate the mobile robots.

The experiment consists of two hunting scenarios. The prey and predator parameters, $\beta = 1.5$, $\gamma = 0.15$, and $v = 0.225$m/s are the same for both scenarios, meaning that the same prey and predators participate in both experiments. These parameters were scaled from simulated examples to appropriate values for the hardware environment. The first scenario, shown in Fig. 7, demonstrates that a single predator is unable to capture the prey. We can verify that

(a)

(b)

Fig. 7. A single robotic "lion" is unable to capture the "gazelle".

$$v = 0.225 < 0.25\sqrt{\frac{0.15}{2e}} \approx 0.25$$

and, therefore, a single robotic lion is not fast enough to capture this particular robotic gazelle.

In the second scenario, shown in Fig. 8, three predators are able to capture the prey together given the same parameters for β, γ, and v, as well as, $\Delta = 0.75$m and $r = 0.75/(2\sin(\pi/8))$. The condition,

$$\Delta = 0.75 > \Delta_x = 0.531 > \sqrt{\frac{0.15}{2}} = 0.274,$$

is satisfied. Fig. 9 illustrates the positions of the predators and prey during the hunt. A color gradient is used to denote the progression of time starting with darker color and ending with lighter color. The prey is not agile enough to escape from the trio of predators; however, it is agile enough to escape a single predator in the first scenario. Such scenarios are often observed in nature when predators (such as lions) sneak up on their prey and the prey is unable to detect the predators early enough and escape. The strategy for capture

(a)

(b)

Fig. 8. Three robotic "lions" are able to cooperatively capture the "gazelle".

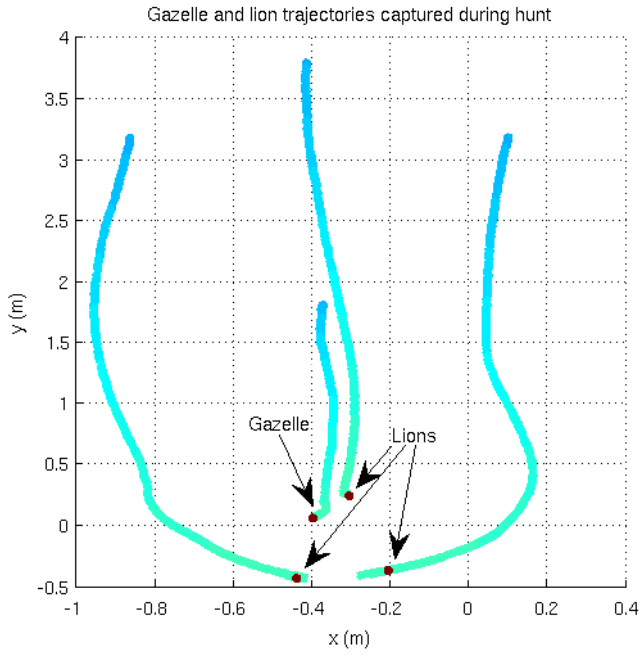

Fig. 9. Recorded positions of a robotic gazelle and three robotic lions during the second scenario.

shown in Fig. 8 is for the wing predators to converge (without collision) in front of the prey once they have achieved crossover and allow the center predator to capture the prey.

V. Conclusion

The experiment validates that we can qualitatively recover hunting strategies from nature and deploy these strategies on mobile multi-agent robot teams. Therefore, we have a valid set of parameters that we can use to quantify the dynamics of the predator-prey interactions and decide whether a particular capture strategy will be successful. This framework can be naturally extended by incorporating additional, more expressive biologically-inspired parameters, such as a maximum evasion velocity for the prey or variations among predators in the group, which correspond to actuator limits, wheel slip, and battery levels on the robotic platforms. More importantly, this type of biologically-inspired parameterization is a useful tool for deriving successful capture strategies for multi-agent robot teams from predator-prey interactions in general.

VI. Acknowledgments

The authors are grateful to Dr. Musad Haque for many helpful discussions on this topic. This work was sponsored by the US Office for Naval Research through the MURI HUNT.

References

[1] T. Balch, "The impact of diversity on performance in multi-robot foraging," in *Proceedings of the third annual conference on Autonomous Agents*. ACM, 1999, pp. 92–99.

[2] F. Belkhouche, B. Belkhouche, and P. Rastgoufard, "Multi-robot hunting behavior," in *Systems, Man and Cybernetics, 2005 IEEE International Conference on*, vol. 3. IEEE, 2005, pp. 2299–2304.

[3] S. Cooper, "Optimal hunting group size: the need for lions to defend their kills against loss to spotted hyaenas," *African Journal of Ecology*, vol. 29, no. 2, pp. 130–136, 1991.

[4] R. Estes, *The behavior guide to African mammals: including hoofed mammals, carnivores, primates*. University of California Press, 1992.

[5] V. Gazi and K. Passino, "Stability analysis of swarms," *IEEE Transactions on Automatic Control*, vol. 48, no. 4, pp. 692–697, 2003.

[6] M. Haque, A. Rahmani, and M. Egerstedt, "Geometric foraging strategies in multi-agent systems based on biological models," in *Decision and Control (CDC), 2010 49th IEEE Conference on*. IEEE, 2010, pp. 6040–6045.

[7] A. T. Hayes, "How many robots? group size and efficiency in collective search tasks," in *Distributed Autonomous Robotic Systems 5*. Springer, 2002, pp. 289–298.

[8] M. Mesbahi and M. Egerstedt, *Graph theoretic methods in multiagent networks*. Princeton University Press, 2010.

[9] R. Olfati-Saber, J. Fax, and R. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[10] K. Pryor and K. Norris, *Dolphin societies: Discoveries and puzzles*. University of California Press, 1998.

[11] B. Pshenichnyi, "Simple pursuit by several objects," *Cybernetics and Systems Analysis*, vol. 12, no. 3, pp. 484–485, 1976.

[12] F. Walther, "Flight behaviour and avoidance of predators in thomson's gazelle (gazella thomsoni guenther 1884)," *Behaviour*, pp. 184–221, 1969.

[13] A. Weitzenfeld, A. Vallesa, and H. Flores, "A biologically-inspired wolf pack multiple robot hunting model," in *Robotics Symposium, 2006. LARS'06. IEEE 3rd Latin American*. IEEE, 2006, pp. 120–127.

[14] H. Yamaguchi, "A cooperative hunting behavior by mobile-robot troops," *The International Journal of Robotics Research*, vol. 18, no. 9, pp. 931–940, 1999.